Introduction: The twin Gravity Recovery and Interior Laboratory (GRAIL) spacecraft were launched in September 2011 on a Discovery-class NASA mission to study the gravitational field of the Moon [1]. Extremely accurate range-rate observations between the two spacecraft at the Ka-band radio wavelength enable the determination of the gravity field of the Moon to very high degree since the data are acquired continuously, even when the spacecraft are not tracked from the Earth. The mission builds on the success of the GRACE (Gravity Recovery and Climate Experiment) to measure the time-variable gravity field of the Earth. The spacecraft flew on a low energy trajectory to the Moon and after orbit insertion, a series of orbit synchroization maneuvers brought the twin spacecraft to the mean mapping orbit altitude of 50 km. The mapping mission for GRAIL commenced on March 1, 2012 and continued until May 29, 2012. During the prime mission, the periastris altitude varied between approximately 20 and 50 km, and the separation distance varied between 80 and 200 kms. The twin spacecraft were tracked in 2-way mode at S Band using the Deep Space Network (DSN). In addition X Band data were collected in 1-way mode using the Ultra-stable Oscillators (USOs) on both of the GRAIL spacecraft.

Approach: We discuss the orbit determination strategies we applied to the analysis of the GRAIL data. While GRACE benefits from homogeneous tracking by the Global Positioning System (GPS) with orbits determined radially to an accuracy of 1-2 cm RMS, the GRAIL orbit determination must rely exclusively on the Deep Space Network (DSN) tracking, and is discontinuous due to occultation while the spacecraft passes over the lunar farside and due to tracking stations not following the GRAIL spacecraft on a continuous basis. In addition, the force model error from the gravity model inaccuracies directly impact the quality of the orbit determination and at the start of the mission easily amounted to tens to a few hundred meters in total position. We characterize the quality of the tracking data over the course of the primary mission, and the resultant orbit performance as the gravity models that we developed were expanded in size from 270x270 with the first month of data, to 360x360 and eventually to 420x420 in spherical harmonics with the full set of primary mission data [2,3]. Models are characterized by the RMS of fit, the orbit determination performance on the GRAIL satellites, and on independent satellites such as Lunar Prospector, and through correlation with detailed models of the lunar topography. The a priori model for the GRAIL POD analyses was the SGM150 (150x150) lunar gravity model based on a combination of SELENE, Lunar Prospector and historical lunar model data[4,5]. The initial fits to the KBRR data were on the order of 1000 – 2000 microns/second. Presently, the fits to the latest 420x420 model are 0.1 to 0.2 microns/sec, except during the low altitude phases where they reach 1-2 microns/sec. Even at these levels, there are indications the signal in the KBRR data residuals has not been exhausted with the 420x420 model, given that the KBRR data quality is estimated to be better than 0.05 microns/second.

We review, compare and contrast the inversion strategies for high degree models and discuss the implementation of the inversion strategies for analysis of GRAIL data on the NASA Center for Climate Simulation (NCCS) supercomputers at NASA GSFC. For the GRACE analyses of time-variable gravity at GSFC we employ a solution scheme relying on the method of preconditioned conjugate gradients. At present for our high-degree GRAIL solutions, we have chosen to rely on direct accumulation of the normal equations in a batch algorithm, adapting our programs to operate efficiently on the NASA GSFC supercomputers. While the orbit determination and preliminary analysis was done on the local work stations, the final accumulations and inversions were performed on the NCCS supercomputers, with a full 420x420 accumulation and inversion taking in general on the order of 12 hrs of wall clock time. We have also developed an inversion strategy based on the use of the square root information filter (SRIF), and we have tested the inversion of GRAIL data using both strategies and identical background modeling to degree 540x540.

The GRAIL orbit determination requires the application of precise orbit and measurement models, and necessitated a number of model improvements in the GEODYN orbit determination program, and how we
validated the KBRR GRAIL data model through analysis of simulated data provided by the GRAIL team at the Jet Propulsion Laboratory.

**Results**: The final model we have developed with the prime mission data is complete to 420x420 in spherical harmonics and is based solely on GRAIL data alone in that no historical data have been included in the solution. We illustrate in Figure 1 the power spectrum of the new 420x420 solution, as well as the a priori 540x540 model, and a solution developed without the application of a Kaula constraint. The a priori model was an attempt to exhaust the signal in the KBRR data, however it still produced anomalous patterns in the gravity anomalies. Thus we updated the parameterization for the modeling of the empirical accelerations in the development of the 420x420 solution GRGM420A. The fact that the power spectrum of the coefficients, and the coefficient standard deviations are an indication that the signal has not been exhausted in the data, and that a higher degree solution should be developed from the data.

![Fig. 1: Power spectrum of the GSFC GRAIL gravity solution, GRGM420A](image_url)

We note that we have calibrated the error covariance using variance component analysis, and at the lower degrees we see an improvement of three orders of magnitude over historical models. In addition to analyzing the power spectra of the new solutions, we evaluate the model using differences with the companion GRAIL model from JPL (GL0420) [1], and through analysis of the power in the Bouguer coefficients derived from a combination of the gravity with topography of the Moon derived from LRO. We also test the solutions through orbit overlaps, and the RMS of fit to the GRAIL data.